ALGEBRA II   Homework 1102   Name:
DR. PAUL L. BAILEY   Tuesday, November 2, 2021

Due Wednesday, November 3, 2021. Write all complex number and polynomials in standard form.

Definition 1. Recall that a polynomial is *monic* if its leading coefficient is 1.

Problem 1. Let $z = 5 + 3i$.

(a) Compute $z + \bar{z}$.

(b) Compute $z\bar{z}$.

(c) Find a monic polynomial f such that $f(z) = 0$.

Problem 2. Solve the following quadratic equations. Write the solution set.

(a) $x^2 + 9 = 0$ (extract the square root)

(b) $x^2 + 8x + 16 = 0$ (its a perfect square)

(c) $x^2 - 5x - 15 = 0$ (factor)

(d) $3x^2 = 7x - 11$ (quadratic formula)

Definition 2. Let f and g be polynomials. We say that f *divides* g, and write $f \mid g$, if there exists a polynomial q such that $g = fq$.

If $f \mid g$, we may say that f is a *factor* of g, or that g is a *multiple* of f.

Proposition 1. (Factor Theorem)
Let $g(x)$ be a polynomial and let $a \in \mathbb{C}$. Let $f(x) = x - a$. Then f is a factor of g if and only if $g(a) = 0$.

Problem 3. Write a monic polynomial f with real coefficients such that $f(1) = 0$ and $f(i) = 0$

Problem 4. Write a polynomial g with integer coefficients such that $g(5) = 0$ and $g\left(\dfrac{2}{3}\right) = 0$.

Problem 5. Let $f(x) = x^5 + 4x^3 - 8x + 3$.

(a) Find the remainder when $f(x)$ is divided by $x - a$, where $a = 2$.

(b) Find the remainder when $f(x)$ is divided by $x - a$, where $a = -1$.

(c) Find the remainder when $f(x)$ is divided by x^2.